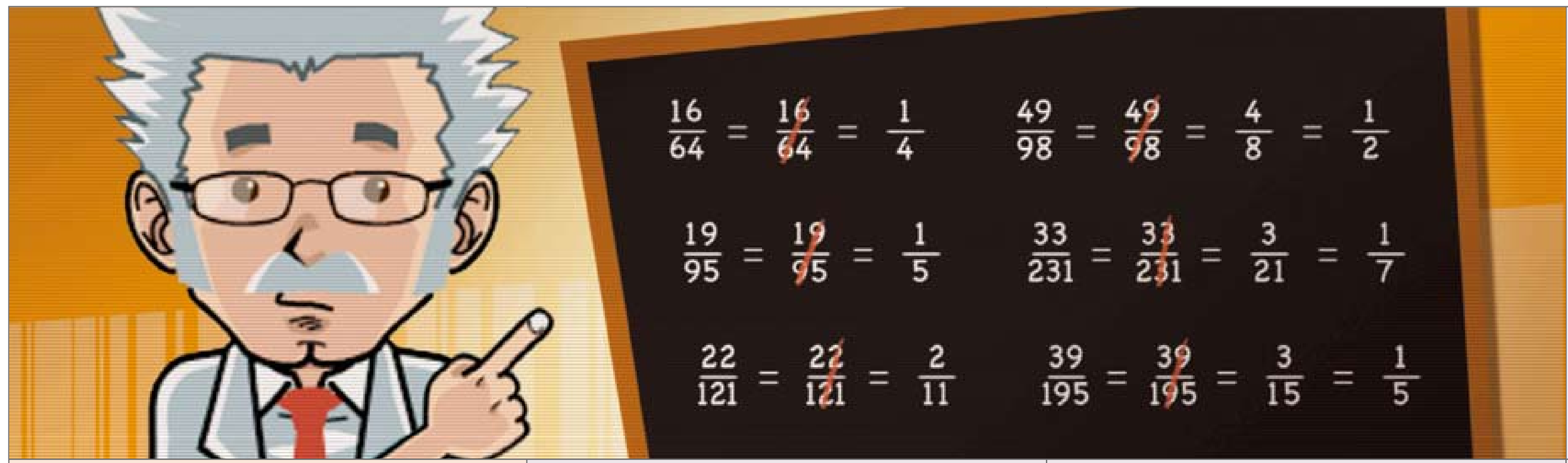
JUN 六月 2011

SUN 日 | MON 一 | TUE 二 | WED 三 | THU 四 | FRI 五 | SAT 六



1 三十

Find $1 + 2 \times 3 \times 5 \times 7 \times 11 \times \ldots \times 97$ (mod 2010)

2 五月

The inequality $kx^2 - (4k + 5)x + 20 < 0$ has ONLY one integral solution 3 and k is an integer. Find k.

3 初二

Given that $x > 0$ and $|x + 1| + |x - 1| = 6$, find x.

4 初三

已知方程$(3 - k)x^2 + kx + 1 = 0$有一正根和一負根，求k的最小整數值。

5 初四

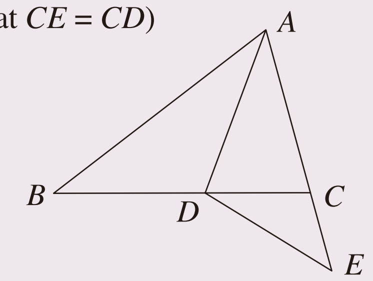
The common ratio of a geometric progression is $\frac{2010 - x}{2001 + x}$; find the minimum integral value of x for which the sum to infinity of the progression exists.

6 端午節 芒種

Find the constant B such that the curve $y = Ax^{\frac{1}{2}} + Bx^{-\frac{1}{2}}$ will have a stationary point (4, 6).

7 初六

In the diagram, AD is an angle bisector of ΔABC. Moreover, $\angle ACB = 2\angle ABC$. It is given that $AB = a^2 - 4b + 4$, $AC = 8c - 27 - 2b^2$ and $CD = 9 + 4a - c^2$. Find $3BC$. (Hint: produce AC to E such that $CE = CD$)



8 初七

a, b, c are non-zero real numbers and $a + b + c \neq 0$. It is given that $\frac{a + b - c}{c} = \frac{c + a - b}{b} = \frac{b + c - a}{a}$. Find the value of $\frac{(a + b)(b + c)(c + a)}{abc}$.

9 初八

At present, Mr. A is 6 times as old as Mr. B. After a few years, Mr. A will be 5 times as old as Mr. B. A few more years later, Mr. A will be 4 times as old as Mr. B. If the present age of Mr. A is $8a$, and a person can live as old as 130 years only, find a.

10 初九

將一粒骰子的六面塗上白色或黑色，問有多少種塗法？

11 初十

Suppose $\sum_{i=1}^{\infty} \frac{i}{k^i} = 0.11$, where $k > 1$. Find k.

12 十一

Find the maximum possible value of $3\left(\sqrt{4a + (b - c)^2} + \sqrt{4b + (c - a)^2} + \sqrt{4c + (a - b)^2}\right)$ where a, b and c are non-negative real numbers such that $a + b + c = 1$.

13 十二

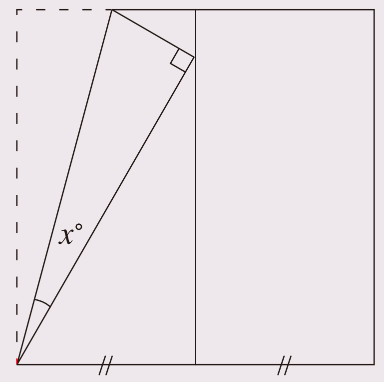
Find $\frac{2}{7}\left(\sin^2(1^\circ) + \sin^2(2^\circ) + \sin^2(3^\circ) + \ldots + \sin^2(90^\circ)\right)$.

14 十三

Birthday of **Andrei Andreyevich Markov**. He was a Russian mathematician. He was best known for his work on theory of stochastic process. Markov chain and Markov process were named after him. His younger brother Vladimir Andreevich Markov was also a mathematician who proved Markov brothers' inequality together with Andrei Markov.

15 十四

A square paper is folded as shown below, find x.



16 十五

Given that for $|x| < 4$, $\sqrt{4 + x} = 2 + \frac{1}{4}x - \frac{1}{4a}x^2 + \ldots$. Find a.

17 十六

The locus of z where z satisfies the relation $|z^{-1} - a| = 4$ is a circle of radius 4 on the complex plane where a is a fixed complex number. Find the value of $|a|^2$.

18 十七

在 $(\sqrt[5]{x} - \frac{1}{x})^n$ 的展開式中，若以x的遞減冪數排列時，第4項是常數，求n。

19 十八

Let $P(A) = \frac{12}{a}$ and $P(B) = \frac{b}{50}$. Given $P(A \cap B) = 0.22$, $P(A' \cap B') = 0.12$, $P(B) - P(A) = 0.14$, find $2a - b$.

20 十九

一個遊戲中，只有一條鑰匙可開啟保險箱門。現盒內共有5條鑰匙，志明和小美輪流隨機抽取一條鑰匙，然後用該鑰匙嘗試開啟保險箱門，誰先開啟保險箱門就可得到箱內所有珠寶。若小美先選擇鑰匙，志明得到珠寶的概率為a，求$50a$的值。

21 二十

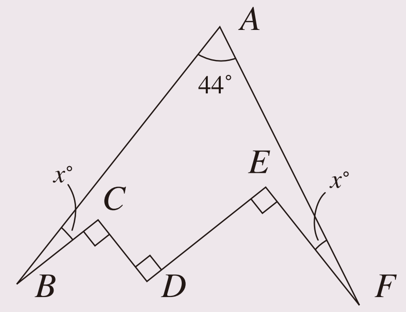
Birthday of **Siméon Denis Poisson**. Among his many mathematical contribution there was a very abstract construct in analytic mechanics (Poisson Brackets, 1809) which helped Dirac formulate a precise correspondence between classical and quantum mechanics.

22 夏至

Find the value of k where $1 \times 1! + 2 \times 2! + \ldots + 21 \times 21! = k! - 1$.

23 廿二

Find the value of x.



24 廿三

How many isosceles triangles with perimeter equals 100 contains 3 sides of integral length?

25 廿四

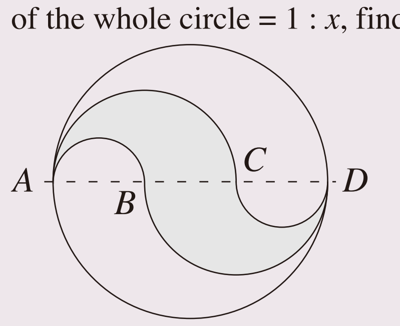
If $|3 + z| = |8 - z|$, find the value of 10 Re(z).

26 廿五

Given that $\sqrt{14} \approx 3.7417$. Find $7\left(\sqrt{8 + 3\sqrt{7}} - \sqrt{8 - 3\sqrt{7}}\right)$, correct to nearest integer, without using calculator.

27 廿六

In the diagram, $AB = BC = CD$. The boundaries of the shaded part are the semi-circles with diameters AB, AC, CD and BD. If area of the shaded part : area of the whole circle = 1 : x, find x^3.



28 廿七

Birthday of **Henri Léon Lebesgue**. He was a French mathematician most famous for his theory of integration, which was originally published in his dissertation *Intégrale, longueur, aire* ("Integral, length, area") at the University of Nancy in 1902.

29 廿八

$\left|\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^{2011} + \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix}^{2011}\right| \equiv k \pmod{31}$ where $1 \le k \le 31$. Find k.

30 廿九

A is the point (7, 5) and B is the centre of the circle $x^2 + y^2 - 2x + 14y - F = 0$. Given that the circle cuts the line segment AB at P, such that $AP : PB = 1 : 2$. Find the value of F.

Miracles do happen

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